an interstage exchange connecting the input nodes to the output nodes,

wherein the interstage exchange is a bit-permuting exchange induced by a permutation σ on integers from 1 to n such that σ maps the numbers $\lfloor n/2 \rfloor+1$, $\lfloor n/2 \rfloor+2$, ..., n, into the set $\{1, 2, ..., \lceil n/2 \rceil\}$ excluding the bit-permuting exchange equal to the $\lfloor n/2 \rfloor^{th}$ power of $SHUF^{(n)}$, and

wherein each $2^k \times 2^k$ generalized divide-and-conquer network (k<n), being representative of each of the input nodes and each of the output nodes, is implemented by forming the bit-permuting 2-stage tensor product, excluding the plain 2-stage tensor product, between a $2^{\lceil k/2 \rceil} \times 2^{\lceil k/2 \rceil}$ generalized divide-and-conquer network and a $2^{\lfloor k/2 \rfloor} \times 2^{\lfloor k/2 \rfloor}$ generalized divide-and-conquer network, recursively until k=1, such that a 2×2 generalized divide-and-conquer network is a single cell.

A4 cont.

11. The $2^n \times 2^n$ generalized divide-and-conquer network as recited in claim 10 wherein the forming of the bit-permuting 2-stage tensor product includes forming a 2-swap tensor product and the bit-permuting exchange is a swap exchange.

12. A $2^n \times 2^n$ generalized divide-and-conquer network, n>3, achieving an optimal layout complexity under the 2-layer Manhattan model with reserved layers and optimal structural modularity among all $2^n \times 2^n$ banyan-type networks, the network comprising

$2^{\lfloor n/2 \rfloor}$ $2^{\lceil n/2 \rceil} \times 2^{\lceil n/2 \rceil}$ input nodes, each of the $2^{\lfloor n/2 \rfloor}$ input nodes being a $2^{\lceil n/2 \rceil} \times 2^{\lceil n/2 \rceil}$ generalized divide-and-conquer network,

$2^{\lceil n/2 \rceil}$ $2^{\lfloor n/2 \rfloor} \times 2^{\lfloor n/2 \rfloor}$ output nodes, each of the $2^{\lceil n/2 \rceil}$ output nodes being a $2^{\lfloor n/2 \rfloor} \times 2^{\lfloor n/2 \rfloor}$ generalized divide-and-conquer network, and

an interstage exchange connecting the input nodes to the output nodes,

wherein each $2^k \times 2^k$ generalized divide-and-conquer network ($k<n$), being representative of each of the input nodes and each of the output nodes, is implemented by forming the bit-permuting 2-stage tensor product, excluding the plain 2-stage tensor product, between a $2^{\lceil k/2 \rceil} \times 2^{\lceil k/2 \rceil}$ generalized divide-and-conquer network and a $2^{\lfloor k/2 \rfloor} \times 2^{\lfloor k/2 \rfloor}$ generalized divide-and-conquer network, recursively until $k=1$, such that a 2×2 generalized divide-and-conquer network is a single cell.

13. The $2^n \times 2^n$ generalized divide-and-conquer network as recited in claim 12 wherein the forming of the bit-permuting 2-stage tensor product includes forming a 2-swap tensor product.

A4 cont

14. A method for constructing a $2^n \times 2^n$ generalized divide-and-conquer network, $n>3$, comprising

determining an n-leaf balanced binary tree indicative of the generalized divide-and-conquer network, $n>3$, and

generating a recursive bit-permuting 2-stage interconnection network, excluding the recursive plain 2-stage interconnection network, associated with the n-leaf balanced binary tree.

15. The method as recited in claim 14 wherein the generating of the recursive bit-permuting 2-stage interconnection network includes generating a recursive 2-swap interconnection network.

16. A method for recursively constructing a $2^n \times 2^n$ generalized divide-and-conquer network, n>3, comprising

forming the bit-permuting 2-stage tensor product, excluding the plain 2-stage tensor product, between a $2^{\lceil n/2 \rceil} \times 2^{\lceil n/2 \rceil}$ generalized divide-and-conquer network and a $2^{\lfloor n/2 \rfloor} \times 2^{\lfloor n/2 \rfloor}$ generalized divide-and-conquer network, and

recursively, each $2^k \times 2^k$ generalized divide-and-conquer network (k<n) is constructed by forming the bit-permuting 2-stage tensor product, excluding the plain 2-stage tensor product, between a $2^{\lceil k/2 \rceil} \times 2^{\lceil k/2 \rceil}$ generalized divide-and-conquer network and a $2^{\lfloor k/2 \rfloor} \times 2^{\lfloor k/2 \rfloor}$ generalized divide-and-conquer network, until k=1, where a 2×2 generalized divide-and-conquer network is a single cell.

A4 cont.

17. The method as recited in claim 16 wherein the forming of the bit-permuting 2-stage tensor product includes forming a 2-swap tensor product.

18. The method as recited in claim 16 wherein each recursive forming of the bit-permuting 2-stage tensor product includes

configuring a first stage of $2^{\lfloor k/2 \rfloor}$ input nodes where each of the input nodes is a $2^{\lceil k/2 \rceil} \times 2^{\lceil k/2 \rceil}$ generalized divide-and-conquer network,

configuring a second stage of $2^{\lceil k/2 \rceil}$ output nodes where each of the output nodes is a $2^{\lfloor k/2 \rfloor} \times 2^{\lfloor k/2 \rfloor}$ generalized divide-and-conquer network, and

interconnecting the first stage and the second stage by a bit-permuting exchange induced by a permutation σ on integers from 1 to k such that σ maps the

numbers $\lfloor k/2 \rfloor+1$, $\lfloor k/2 \rfloor+2$, …, k, into the set $\{1, 2, \ldots, \lceil k/2 \rceil\}$ excluding the bit-permuting exchange equal to the $\lfloor k/2 \rfloor^{th}$ power of $SHUF^{(k)}$.

19. The method as recited in claim 18 wherein the interconnecting the first stage and the second stage by a bit-permuting exchange includes forming the bit-permuting exchange as a swap exchange.

20. The method as recited in claim 16 wherein each recursive forming of the bit-permuting 2-stage tensor product between a $2^{\lceil j/2 \rceil} \times 2^{\lceil j/2 \rceil}$ generalized divide-and-conquer network and a $2^{\lfloor j/2 \rfloor} \times 2^{\lfloor j/2 \rfloor}$ generalized divide-and-conquer network, $1 \leq j \leq n$, includes

A4 Cont.

configuring a first stage of $2^{\lfloor j/2 \rfloor}$ input nodes where each of the input nodes is a $2^{\lceil j/2 \rceil} \times 2^{\lceil j/2 \rceil}$ generalized divide-and-conquer network,

configuring a second stage of $2^{\lceil j/2 \rceil}$ output nodes where each of the output nodes is a $2^{\lfloor j/2 \rfloor} \times 2^{\lfloor j/2 \rfloor}$ generalized divide-and-conquer network, and

interconnecting the first stage and the second stage by a bit-permuting exchange induced by a permutation σ on integers from 1 to j such that σ maps the numbers $\lfloor j/2 \rfloor+1$, $\lfloor j/2 \rfloor+2$, …, j, into the set $\{1, 2, \ldots, \lceil j/2 \rceil\}$ excluding the bit-permuting exchange equal to the $\lfloor j/2 \rfloor^{th}$ power of $SHUF^{(j)}$.

21. The method as recited in claim 20 wherein the interconnecting the first stage and the second stage by a bit-permuting exchange includes forming the bit-permuting exchange as a swap exchange.

22. A method for recursively constructing a $2^n \times 2^n$ generalized divide-and-conquer network, n>3, in correspondence to an n-leaf balanced binary tree, the method comprising

constructing, in correspondence to the root R of the tree, the $2^n \times 2^n$ generalized divide-and-conquer network by forming the bit-permuting 2-stage tensor product between a $2^p \times 2^p$ generalized divide-and-conquer network which is associated with the left-son of R having a weight of p and a $2^q \times 2^q$ generalized divide-and-conquer network which is associated with the right-son of R having a weight of q, with $|p-q| \leq 1$ and wherein $p = \lceil n/2 \rceil$ and $q = \lfloor n/2 \rfloor$, or $p = \lfloor n/2 \rfloor$ and $q = \lceil n/2 \rceil$, and

A4 cont.

recursively, in correspondence to a generic internal node H with weight k (k<n) until k=1 and wherein a 2×2 generalized divide-and-conquer network is a single cell, constructing a $2^k \times 2^k$ generalized divide-and-conquer network by forming the bit-permuting 2-stage tensor product between a $2^s \times 2^s$ generalized divide-and-conquer network which is associated with the left-son of H having a weight of s and a $2^t \times 2^t$ generalized divide-and-conquer network which is associated with the right-son of H having a weight of t, with $|s-t| \leq 1$ and wherein $s = \lceil k/2 \rceil$ and $t = \lfloor k/2 \rfloor$, or $s = \lfloor k/2 \rfloor$ and $t = \lceil k/2 \rceil$.

23. The method as recited in claim 22 wherein the forming of the bit-permuting 2-stage tensor product includes forming a 2-swap tensor product.--